Written Exam for the M.Sc. in Economics 2010-II

Advanced Industrial Organization

Final Exam

8 June, 2011

(24 hour take home exam)

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ALL QUESTIONS BELOW SHOULD BE ANSWERED

1. **Problem 1.**

   We consider a differentiated Hotelling market. A continuum of consumers are located on a line of length one. Consumer x is located in $x \in [0, 1]$. Two firms are located at the ends of the line, firm A in zero and firm B in 1. Both firms have constant marginal costs normalized to zero. In each period, a consumer buys at most one unit of the (differentiated) good. If she buys at the price p from a firm, located d away from her, her utility is

   $$V = u - p - td, \tag{1}$$

   where $u > 0$ is the reservation price, and $t > 0$ is the transportation cost, reflecting the consumer's "pickiness". We assume that $2u > t$, so that the consumer in the middle is not excluded even at zero prices. Firms are supposed to maximize profits and set prices simultaneously.

   (a) *Find the Nash equilibrium, equilibrium prices, profits and demand for each firm.*

   Standard Hotelling model, the result is $p = t$ and $x \leq 1/2$ buys from A and $x = 1/2$ buys from B. Firms profit is $\pi = t/2$. The solution is efficient, consumers' transportation costs are minimized.

   (b) *Now suppose the line [0,1] is divided into four equally large regions: I ,II, III, and IV. Region I is the line segment [0,1/4[, region II [1/4,1/2[, Region III [1/2,3/4[and region IV [3/4,1]. The firms do not know the exact locations of the consumers. However, imagine that the regions each have a particular zip-code (postnummer) and the firms can send out different coupons to the different regions, thus recognizing which region a costumer belongs to. So if they send out coupons, they will be able to learn which region a consumer belongs to when she arrives at the firm.*

   Find the Nash equilibrium,equilibrium prices, profits and demand for each firm.

Notice that region I and IV are symmetric and region II and III are symmetric, hence we only need to consider regions I and II

Consider region I. Let p_{AI} and p_{BI} denote the prices of firms A and B in the region, respectively.

The indifferent consumer is located in

$$x = \frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}$$

(this formula is only valid as long as the solution $x \in [0, \frac{1}{4}]$, but as we will see below this is fulfilled in the eq we derive. The completely correct way to do things is to note that the indifferent consumer is located either in x given by the formula, 1/4 (if the formula gives a larger result) , or 0 (if the formula gives a lower result). In fancy math this can be written

$$x = \min\left[\max\left[0, \frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}\right], \frac{1}{4}\right] \tag{FANCY}$$

But we do not need this (now). Firm $A's$ profit in region I

$$p_{AI}\left(\frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}\right)$$

the best reply

$$\max_{p_{AI}} \; p_{AI}\left(\frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}\right)$$

gives foc

$$\left(\frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}\right) - \frac{p_{AI}}{2t} = 0$$

so that

$$p_{AI} = \frac{1}{2}(t + p_{BI}) \tag{BR A Region I}$$

Recall that region I consists of $x \in [0, 1/4]$, hence firm $B's$ demand comes from those to the right of the indifferent consumer and to the left of 1/4. Therefore we can write firm B's profit as

$$p_{BI}\left(\frac{1}{4} - \left(\frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}\right)\right)$$

foc

$$\left(\frac{1}{4} - \left(\frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t}\right)\right) - \frac{p_{BI}}{2t} = 0$$

and the solution is:

$$p_{BI} = \frac{1}{2}p_{AI} - \frac{1}{4}t$$

So the equilibrium on turf I solves the two equations (best replies)

$$\begin{aligned} p_{AI} &= \frac{1}{2}(t + p_{BI}) \\ p_{BI} &= \frac{1}{2}p_{AI} - \frac{1}{4}t \end{aligned}$$

which gives the solution

$$p_{AI} = \frac{1}{2}t, p_{BI} = 0$$

The indifferent consumer in region I is therefore located at

$$\frac{1}{2} + \frac{p_{BI} - p_{AI}}{2t} = \frac{1}{2} + \frac{0 - \frac{1}{2}t}{2t} = \frac{1}{4}$$

I.e in the rightmost point of the region. Hence everybody buys from firm A.

In region I firm A earns

$$\pi_{AI} = p_{AI} \cdot \frac{1}{4} = \frac{1}{2}t\frac{1}{4} = \frac{1}{8}t$$

and firm B earns nothing.

Consider region II

Firm $A's$ profit in region II

$$p_{AII}\left(\frac{1}{2} + \frac{p_{BII} - p_{AII}}{2t} - \frac{1}{4}\right)$$

the best reply

$$\max_{p_{AII}} \; p_{AII}\left(\frac{1}{2} + \frac{p_{BII} - p_{AII}}{2t} - \frac{1}{4}\right)$$

gives foc

$$\left(\frac{1}{2} + \frac{p_{BII} - p_{AII}}{2t} - \frac{1}{4}\right) - \frac{p_{AII}}{2t} = 0$$

,so that

$$p_{AII} = \frac{1}{4}t + \frac{1}{2}p_{BII} \qquad \text{(BR A Region II)}$$

Firm B

$$p_{BII}\left(\frac{1}{2}-\left(\frac{1}{2}+\frac{p_{BII}-p_{AII}}{2t}\right)\right)$$

foc

$$\left(\frac{1}{2}-\left(\frac{1}{2}+\frac{p_{BII}-p_{AII}}{2t}\right)\right)-\frac{p_{BII}}{2t}=0$$

,so

$$p_{BII}=\frac{1}{2}p_{AII}$$

so the equilibrium in region II is

$$\begin{aligned}p_{AII} &= \frac{1}{4}t+\frac{1}{2}p_{BII}\\ p_{BII} &= \frac{1}{2}p_{AII}\end{aligned}$$

so the equilibrium is

$$p_{AII}=\frac{1}{3}t, p_{BII}=\frac{1}{6}t$$

and the indifferent consumer is located at

$$\frac{1}{2}+\frac{p_{BII}-p_{AII}}{2t}=\frac{1}{2}+\frac{\frac{1}{6}t-\frac{1}{3}t}{2t}=\frac{5}{12}$$

In region II firm A earns

$$\pi_{AII}=p_{AII}\cdot\left(\frac{5}{12}-\frac{1}{4}\right)=\frac{1}{3}t\left(\frac{5}{12}-\frac{1}{4}\right)=\frac{1}{18}t$$

while the profit of firm B is

$$\pi_{BII}=p_{BII}\cdot\left(\frac{1}{2}-\frac{5}{12}\right)=\frac{1}{6}t\left(\frac{1}{2}-\frac{5}{12}\right)=\frac{1}{72}t$$

By symmetry firm A earns the same in region III as firm B does in region II and the same in region IV as firm B does in region I. Hence, the total profit of firm A is

$$\pi_{AI}+\pi_{AII}+\pi_{AIII}+\pi_{IV}=\frac{1}{8}t+\frac{1}{18}t+\frac{1}{72}t+0=\frac{7}{36}t$$

By symmetry firm B earns the same profit.

c. *Compare with the solution found in a. Discuss whether the knowledge of which regions consumers belong to is advantageous for the firms, for welfare and for consumer welfare. If you should advise a consumer agency in its lobby activities would you endorse coupons?*

1. (a) It is obvious from the profits that the use of cupons hurt hte firms profits $\frac{7}{36}t < \frac{1}{2}t$.

   Furthermore coupons induce an inefficiency since consumers in region II to the left of $1/2$ and to the right of $5/12$ buy from firm B, their transportation cost is larger than in the standard Hotelling eq in question a. Similarly, consumers in region III to the left of $7/12$ buy from firm A incurring larger transportation cost than in the Hotelling equilibrium.

   However, the equilibrium is beneficial for consumers. This can be shown in two ways. Calculate the utility of each consumer and integrate over x. Alternatively, realize that all consumers face lower prices, when coupons are used than when they are not. Furthremore, consumers in regions II and III has the option to buy from firm A (B) at lower prices than in the Hotelling equilibrium. This would make them better off than in the Hotelling equilibrium. In fact, they prefer to buy from the other firm, this must be because they are even better of then. Hence, even though some consumers incur higher transporation costs the low prices dominate from the consumers point of view.

   In conclusion, if you advise a consumer agency - speaking about this market $-$ you should endorse coupons.

d. *Suppose the firms agree not to use coupons? Is this a credible agreement that can be sustained in equilibrium?*

The answer is no. To see it suppose that firm B refrains from using coupons charging the same price p_B on the whole line. Look at firm $A's$ best replies (BR A Region I) and ($BR\ A\ RegionII$). They give that firm $A's$ best replies in the two regions differ

$$p_{AI} = \frac{1}{2}(t + p_B) \neq p_{AII} = \frac{1}{4}t + \frac{1}{2}p_B$$

Hence, given that firm B does not use Coupons to distinguish between the regions, it would be beneficial for firm A to do.

e. *Suppose now each period described above is repeated an infinitely number of times, so that time runs from* $t = 0, ..., \infty$. *Firms discount future profits with the discount factor* δ, *where* $0 < \delta < 1$. *Firms seek to maximize the discounted sum of profits. Show that there is a trigger strategy equilibrium, where in the normal phase the firms refrain from using coupons and realize the monopoly profit and in the trigger strategy phase play an infinite repetition of the one-shot Nash equilibrium, where they use coupons.*

The question merges different parts of the curriculum. A simple and okay way to answer it is to notice first that the per firm profit not using coupons $\pi_{NC} = \frac{1}{2}t > \pi_C = \frac{7}{36}t$. Secondly, the profit from deviation from not using coupons $\pi_D > \pi_{NC}$ as we know from question d.

A trigger strategy equilibrum sustaining non use of coupons takes that the gain from deviation $\pi_D - \pi_{NC}$ is smaller than the future loss from entering the punishment phase $\frac{\delta}{1-\delta}(\pi_{NC} - \pi_C)$. Hence if the discount factor is sufficiently high so that

$$\pi_D - \pi_{NC} \leq \frac{\delta}{1-\delta}(\pi_{NC} - \pi_C)$$

i.e

$$\delta \geq \frac{\pi_D - \pi_{NC}}{\pi_D - \pi_C}$$

Now of couse it is in this example possible to calculate the crucial discount factor but **the question does not ask for it**.

Given the way the question is asked it is in fact an okay answer to say, that this needs not necessarily be true. It takes that the discount factor is suffiently high, as demonstrated above.

1 Empirical Question

1. This set of questions are about the work in Porter's 1983 paper "A Study of Cartel Stability: The Joint Executive Committee, 1880–1886." In this paper, Porter uses data from a railroad cartel to empirically test the proposition that observed prices reflected switches from collusive to noncooperative behaviour. To answer these questions, you need to first download the data set *RailData.xls* from the course webpage.

   (a) Porter assumes that aggregate demand is a log-linear function of prices:

   $$\log(Q_t) = \alpha_0 + \alpha_1 \log(p_t) + \alpha_2 L_t + U_{1t}$$

   where $\{U_{11}, U_{12}, \ldots, U_{1T}\}$ is a sequence of *i.i.d* normal random variables—$U_{1t} \sim N(0, \sigma_1^2)$. There are N firms in the industry each with a different cost function:

   $$C_i(q_{it}) = a_i q_{it}^{\delta} + F_i.$$

   Given these assumed functional forms, the market share of firm i in period t is

   $$s_{it} = \frac{a_i^{\frac{1}{1-\delta}}}{\sum_j a_j^{\frac{1}{1-\delta}}}.$$

   Assume that the switches between collusive and noncooperative behaviour are known (this is the PO data). The estimable supply relationship that Porter estimates (equation 2 in the paper) is

   $$\log(p_t) = \beta_0 + \beta_1 \log(Q_t) + \beta_2 S_t + \beta_3 I_t + U_{2t}.$$

   Porter also includes monthly dummy variables in the empirical specification. Estimate this equation using **OLS**. Report the estimated values of $\beta_0, \beta_1, \beta_3$. Compare your estimate of β_1 with the one reported in column 3 of table 3. Provide an explanation for the difference between the two estimates.

   SOLUTION: The estimate of the parameters are: $(\beta_0, \beta_1, \beta_3) = (-0.814, -0.126, 0.267)$. Porter's estimate of β_1 is 0.251. There is clearly a big difference. Porter's estimates indicate that prices are increasing in the quantity of grain shipped, whereas the OLS estimates indicate that prices are a decreasing function of the quantity of grained shipped. The problem with estimating the supply curve using OLS is that there is a simultaneity problem in the regression function. That is, price and quantity are determined simultaneously as equilibrium outcomes. Therefore, the quantity variable is correlated with the error term and the estimates are inconsistent. This is why the supply equation must be estimated using some variant of two-stage least squares. In particular, one must find an instrument for quantity. A variable that shifts the demand curve but is uncorrelated with the error term.

   (b) Estimate the demand and the supply equations specified by Porter using two-stage least squares. Report your estimates. How close are they to the estimates reported in columns 2 and 3 in table 3? Use your results to obtain an estimate of the behaviour parameter θ.

   SOLUTION: The estimates are reported in the table.

Variable	Supply	Demand
q	.251	
po	.362	
dm1	-.201	
dm2	-.172	
dm3	-.322	
dm4	-.208	
Lakes		-.437
Constant	-3.944	9.169
Price		-.742

The estimates are almost identical to those of Porter. If we assume that $\beta_3 = -\log(1 + \frac{\theta}{\alpha_1})$, then we can use the estimates of β_3 and α_1 to calculate the value of θ. The implied value of θ is 0.225.

(c) Suppose that the estimated price elasticity of demand is the true elasticity. Test the null hypothesis that the collusive regime sets static monopoly prices.

SOLUTION: If we assume that our estimated elasticity of demand is the true elasticity then we can treat α_1 as a number and not worry about any issues with estimation error. Under the assumption that firms set static monopoly prices $\theta = 1$ and $\beta_3 = -\log(1 + \frac{1}{\alpha_1})$. We can use these results to form the test statistic:

$$T = \alpha_1 - \frac{1}{\exp(-\beta_3) - 1}.$$

Under the null hypothesis, $T = 0$. The value of the test statistic is 2.5419 with a standard error of 0.389. Therefore, the null hypothesis of static monopoly pricing can be rejected.

(d) Consider an alternative specification. Assume that marginal costs are constant:

$$C = aq_i.$$

In addition, assume that aggregate demand is linear:

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 L_t + \alpha_3 Seas_t + U_{1t}.$$

In addition, suppose that you as the econometrician have no information concerning when firms are in a collusive phase or not. So, in this case you do **NOT** observe the variable PO. Derive the supply relationship under the assumption of joint profit maximization. Given the functional forms and the new assumptions, is θ identified? That is, can you use two-stage least squares to estimate θ? SOLUTION: Perceived marginal revenue for is

$$\theta \frac{\partial P}{\partial Q} Q + P = \theta \frac{1}{\alpha_1} Q + P.$$

The generalized supply relationship is

$$P = a - \theta \frac{1}{\alpha_1} Q.$$

Under the assumption that firms are joint profit maximizers $\theta = 1$, so

$$P^m = a - \frac{1}{\alpha_1} Q.$$

A simple specification for estimating the supply equation is

$$P^m = \beta_0 - \beta_1 Q_t + U_{2t}.$$

Note that θ is identified since we can obtain an estimate of β_1 and and estimate of α_1: θ equals the product of the two estimates.

(e) Estimate your supply equation and demand equation using two-stage least squares. Be sure to report the the instruments that you use for the endogenous variables in both equations. Report your estimates. Calculate the implied value of θ. How does it compare with the value you calculated in part b)? Provide an explanation of why the two estimates are different.

SOLUTION: The implied value of θ is 0.080 which is smaller than the value found in part b). One reason that it is smaller is that the two models are different. Another reason is that in the current model, since we are not controlling for switches, θ is not the conduct parameter during collusive phases, but should be interpreted as the average conduct parameter over all phases. So, it should be lower.

(f) Calculate the estimated elasticity of demand. Evaluate the elasticity at the sample means. How does it compare to Porter's estimate?

SOLUTION: The price elasticity of demand is -0.62 which is a little larger than Porter's estimate which equals -0.742.

Consider a market with two firms, firm 1 and firm 2, producing a homogenous good at a constant marginal cost of c, $c > 0$. The demand for the product is given by a linear demand curve $D(p) = A - p$, where $A > 0$. The firms compete a la Bertrand.

(a) Find the price, total quantity sold and the firms' profits in the Bertrand equilibrium?
Price = c, Quantity = $A - c$, Profits = 0 (answers with ≈ or +/- ε are okay)

The production costs may be reduced by a cost saving innovation.

To have the possibility to obtain this innovation a firm must invest I in research and development. If one of the firms invests in research and development the probability that the innovation is made is p_1. If both firms invest in research and development, the innovation is obtained with probability p_2, where it is assumed that $p_1 < p_2 < 2p_1$. If a firm has access to the innovation, its production cost is lowered to $c - d$, where $0 < d < c$.

(b) Suppose that the innovation is not patentable and can be freely and costlessly copied. Will the firms invest in research and development?
No, profits will still be zero after innovation, so investment cannot be justified

The innovation is patentable, and if one of the firms holds the patent, it can produce at the low cost $c - d$, whereas its competitor has to stick to the old technology and production cost c.

(c) Find the price, total quantity sold and the firms' profits in the Bertrand equilibrium, when firm 1 holds the patent.
Price = c, Quantity = $A - c$, Profit = $d(A - c)$ to firm 1 (answers with ≈ or +/- ε are okay)

Suppose that the patent and the market run infinitely, such that research and development investments are made at $t = 0$, and if a firm obtains a patent, it will get the profit calculated in (c) for $t = 1, 2, 3 \ldots$. If both firms invest in research and development, they each have the same probability of obtaining the patent. Future profits are discounted by a factor δ, $0 < \delta < 1$.

(d) Suppose that firm 1 knows that firm 2 *does not* invest in research and development. For which values of I does it pay for firm 1 to invest in research and development at $t = 0$.
$I \leq p_1 d(A - c) \delta/(1 - \delta)$

(e) Suppose that firm 2 knows that firm 1 *does* invest in research and development. For which values of I does it pay for firm 2 to invest in research and development at $t = 0$.
$I \leq ½ p_2 d(A - c)\delta/(1 - \delta)$

Now assume that none of the firms can observe whether the other firm invests in research and development before it decides whether to invest itself. Consider the game played by the two firms at $t = 0$.

(f) For which values of I is there a Nash-equilibrium in the game in which none of the firms invest in research and development?
$I \geq p_1 d(A - c)\delta/(1 - \delta)$

(g) For which values of I is there a Nash-equilibrium in the game in both firms invest in research and development?
$I \leq ½ p_2 d(A - c) \delta/(1 - \delta)$

(h) For which values of I is there a Nash-equilibrium in the game in which firm 1 invests in research and development, but firm 2 does not?
$p_1 d(A - c) \delta/(1 - \delta) \geq I \geq ½ p_2 d(A - c) \delta/(1 - \delta)$

Now consider a social planner who can decide the research and development of the firms, but has to respect the patent system and cannot regulate production or pricing.

(i) Find as a function of I, the number of firms (0, 1 or 2) that the planner will decide should invest in research and development. Compare to (f) - (h) above.
$I \leq (p_2 - p_1) d(A - c) \delta/(1 - \delta)$ it is efficient that 2 firms invest
For $p_1 d(A - c) \delta/(1 - \delta) \geq I \geq (p_2 - p_1) d(A - c) \delta/(1 - \delta)$ it is efficient that 1 firm invests
For $I \geq p_1 d(A - c) \delta/(1 - \delta)$ it is efficient that 0 firms invest
For $½ p_2 d(A - c) \delta/(1 - \delta) \geq I \geq (p_2 - p_1) d(A - c) \delta/(1 - \delta)$ the game has an equilibrium in which both firms invest, but this is not efficient. The problem is that firm 2 does not take into account that if it invests, then firm 1's expected profit decreases. Other equilibria in (f)-(h) are efficient.
(not required) In case $p_1 d(A - c) \delta/(1 - \delta) \geq I \geq ½ p_2 d(A - c) \delta/(1 - \delta)$, the game also has mixed strategy equilibria, which are not efficient.

(j) Suppose a firm that holds the patent can license it to the competitor. Will the firm do so? Will the possibility of licensing influence the analysis in (d)-(i) above?
It the firm requires a licensing fee of size d, then it is possible that the competitor will buy it, and produce. The analysis will otherwise be unchanged, in particular the patent holder will obtain the same profit as in (c) independent of how the production is distributed among the two firms.

From now on assume that firm 2 never invests in research and development, and consider the case $A = 100$, $c = 30$, $d = 10$, $I = 2000$, $p_1 = 0.4$, and $\delta = 0.9$.

Suppose that the patent is not infinite, but has a *limited duration* T, such that if firm 1 obtains the patent, it has a cost advantage over firm 2 for $t = 1, \ldots, T$, but firm 2 can freely and costlessly use the innovation from period $T + 1$ and onwards.

(k) What is the minimal duration of the patent that will ensure that firm 1 will invest in research and development?
$T \geq$ ln(0.9-5/7)/ln0.9 -1 = 14,98, i.e., the minimal duration is 15 periods.

(l) Explain that there is a welfare loss incurred in each of the *T* periods compared to a situation when the innovation has been made, but not patented. Calculate the total size of this welfare loss assuming that social welfare is discounted by the same factor as profit, i.e., $\delta = 0.9$.
Efficient production requires price = marginal costs ($c - d$), but with the patent price exceeds costs by the profit margin *d*. The total deadweight loss is ½ $d^2 = 50$ each period. The total present value of the deadweight loss for 15 periods discounted to the present ($t = 0$), is $50 \cdot 10(0.9 - 0.9^{16}) = 357.35$.

Now suppose that the patent is infinite, but has a *limited breadth b*, $0 < b < 1$, such that if firm 1 holds the patent firm 2 can partly copy the technology and produce at marginal costs $c - d(1\text{-}b)$.

(m) Explain briefly how the breadth of a patent is determined. How may patent authorities influence the breadth?
Possibility for competitor to produce similar products, e.g., by inventing around, reverse engineering, compulsory licensing at low fees.
Number of clauses, extension of clauses.

(n) Find the price, total quantity sold and the firms' profits in the Bertrand equilibrium when firm 1 holds the patent.
Price = $c - d(1 - b) = 20 + 10b$, Quantity = $A - c + d(1 - b) = 80 - 10b$, Profit = $(A - c + d(1 - b))db = 800b - 100b^2$ to firm 1 (answers with ≈ or +/- ε are okay)

(o) What is the minimal breadth of the patent that will ensure that firm 1 will invest in research and development (assuming that the patent is infinite)?
$b \geq 4 - \sqrt{94/3} = 0,768$

(p) Explain that there is a welfare loss incurred in each period compared to a situation when the innovation has been made, but not patented. Calculate the total size of this welfare loss assuming that social welfare is discounted by the same factor as profit, i.e., $\delta = 0.9$.
Price exceeds costs by $10b$. Welfare loss per period $50b^2$. Total welfare loss $450b^2 = 265.57$.

Metode ser ok ud. Mangler mellemregninge

(q) Compare the welfare losses found in (l) and (p) and comment.
This is a case where length is better than breadth in providing sufficient incentives (long patents best), since increasing the breadth increases the welfare loss at an increasing rate.